RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

SECOND YEAR

B.A./B.SC. THIRD SEMESTER (July – December), 2011

Mid-Semester Examination, September, 2011

Date : 12/09/2011

MATHEMATICS (Honours)

Time : 2 pm – 4 pm

Paper : III

Full Marks : 50

(Use separate answer scripts for each group)

Group - A

1. Answer **any two** :

a) If $ax^2 + 2hxy + by^2$ be transformed to $a'x'^2 + 2h'x'y' + b'y'^2$ by the transformation of rotation, show that— i) $a' + b' = a + b$

ii) $a'b' = h'^2 = ab - h^2$ [5]

b) Find the equations of the projection of the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ on the plane $x + 2y + z = 6$. [5]

c) A variable plane has intercepts on the coordinate axes, the sum of whose squares is a constant K^2. Show that the locus of the foot of the perpendicular from the origin to the plane is—
$(x^2 + y^2 + z^2)(x^{-2} + y^{-2} + z^{-2}) = K^2$. [5]

d) Find the length and the equations of the S.D. between the lines—
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}; \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ [5]

Group – B

Answer either question no. 2 or question no. 3

2. a) A weightless elastic string of natural length ℓ and modulus λ, has two equal particles of mass m at its ends and lies on a smooth horizontal table perpendicular to an edge with one particle just hanging over. Show that the other particle will pass over at the end of time t given by the equation—

$$2\ell + \frac{mg\ell}{\lambda}\sin^2\sqrt{\frac{\lambda}{2m\ell}}t = \frac{1}{2}gt^2$$ [8]

b) A particle is projected with velocity u at an inclination α above the horizontal in a medium whose resistance per unit mass is K times the velocity. Show that its direction will again make angle α below the horizontal after a time $\frac{1}{K}\log\left(1+\frac{2Ku}{g}\sin\alpha\right)$. [7]

3. a) If h be the height attained by a particle when projected with a velocity V from the earth's surface supposing its attraction constant, and H the corresponding height when the variation of gravity is taken into account, prove that $\frac{1}{h} - \frac{1}{H} = \frac{1}{r}$ where r is the radius of the earth. [7]

b) A particle describes a rectangular hyperbola, the acceleration being directed from the centre. Show that the angle θ described about the centre in time t after leaving the vertex is given by the equation $\tan\theta = \tan h(\sqrt{\mu}t)$ where μ is the acceleration at distance unity. [8]

Group – C

4. Answer **any five** questions :

a) Let V and W be two vector spaces over a field F and let $T:V\to W$ be a linear transformation. Define Ker T and show that Ker T is a subspace of V. Obtain a sufficient condition under which T will be injective. [1+2+2]

b) A linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ transforms the vectors (1,0,0), (1,1,0), (1,1,1) to the vectors (1,3,2), (3,4,0) and (2,1,3) respectively. Find T and matrix representation of T relative to the standard ordered basis of $\mathbb{R}^3$. [2+3]

c) i) Let V and W be vectors spaces over a field F. Let $T:V\to W$ be a linear transformation. Prove that image of T is a subspace of W. [2]

ii) A linear transformation $T:\mathbb{R}^3\to\mathbb{R}^4$ is defined by $T(x_1, x_2, x_3) = (x_2 + x_3, x_3 + x_1, x_1 + x_2, x_1 + x_2+x_3); (x_1,x_2,x_3)\in\mathbb{R}^3$ Find Ker T. What conclusion can you draw regarding the linear dependence / independence of the image set of the set of vectors {(1,0,0), (0,1,0), (0,0,1} of $\mathbb{R}^3$? [2+1]

d) The matrix of a linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ with respect to the ordered basis {(0,1,1), (1,0,1), (1,1,0)} of $\mathbb{R}^3$ is given by $\begin{pmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{pmatrix}$. Find the matrix of T relative to the ordered basis {(2,1,1), (1,2,1), (1,1,2)}. [5]

e) Let V and W be vector spaces over a field F and V be finite dimensional. If $T:V\to W$ be a linear transformation, prove that nullity of T + rank of T = dim V. [5]

f) Let V be a vector space of dimension m and W be a subspace of V of dimension n over the field F. Prove that dim $\frac{V}{M}=m-n$. [5]

g) Let V and W be finite dimensional vector spaces over a field F and $T:V\to W$ be a linear transformation. Prove that rank of T = rank of matrix of T. [5]

h) When are two vector spaces called isomorphic? A linear mapping $T:\mathbb{R}^3\to\mathbb{R}^3$ maps the vectors (2,1,1), (1,2,1), (1,1,2) to (1,1,–1), (1,–1,1), (1,0,0) respectively. Examine whether T is an isomorphism. [1+4]